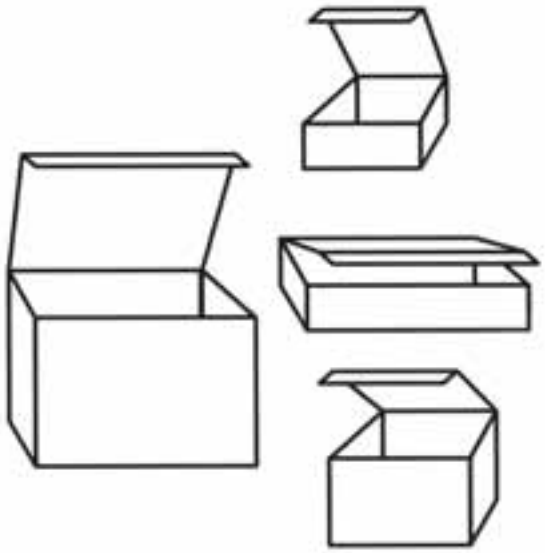
What's the Best Package?

Manufacturers often face the difficulty of packing items into containers that are a different shape than what is being packed. They must design the least expensive packaging arrangement that reduces the risk of breakage. In other words, the items must be securely packaged in a carton with dimensions that require the least amount of cardboard.

Suppose the contents are shaped like the TesselTiles™ orange right triangle. What container would require the least amount of surface area?



Topic of Investigation

Surface Area of Rectangular Prisms

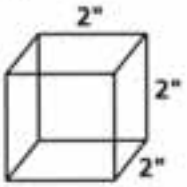
Questions to Investigate

- *What are the general properties and parts of rectangular prisms?*
- *Which arrangement of orange TesselTiles would produce the rectangular prism having the least amount of surface area?*

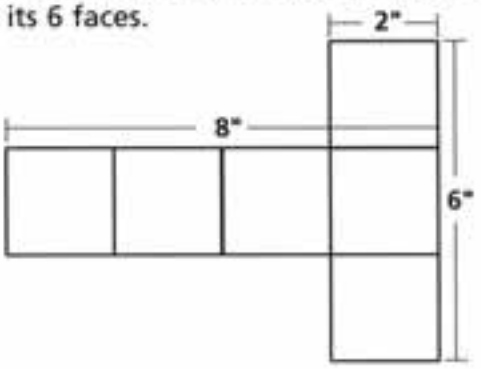
Mathematical Review

Surface area is the sum of the areas of all faces of a three-dimensional figure or polyhedron.

For example: the surface area of this cube that is made up of 2-inch sides...



...can be found by adding the areas of each of its 6 faces.

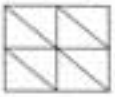


Predict the Results

The Sweet Acre Candy Company is releasing a new line of candy. The candy is the shape and size of the TesselTiles™ orange right triangle. The Sweet Acre Candy Company wants to manufacture a box in the shape of a rectangular prism that will hold 24 candies. To save money, they are interested in manufacturing a box that will use the least amount of cardboard.

Without measuring, rank the candy arrangements shown in the Predictions Chart according to which arrangement would yield a rectangular prism with the smallest surface area, thus requiring the least amount of cardboard. Enter your rankings to the right of each sketch. Use "1" to label the arrangement requiring the *least* amount of cardboard, through "11" for that which requires the *greatest* amount.

Predictions Chart: Packaging Arrangements for 24 Orange Right Triangles

Candy Layers	Top View of Package Arrangements	Rank the Total Surface Area
Single	1 	
	2 	
	3 	
	4 	
Double	5 	
	6 	
	7 	
Triple	8 	
	9 	
Quadruple	10 	
	11 	

Procedure

1. Select 24 orange right triangles from your TesselTiles™ set. Use your TesselTiles to construct the arrangements shown in the Predictions Chart. When constructing different rectangular-shaped boxes to hold your 24 candies, consider

- the arrangement of the candies;
- the number of candy layers;
- the surface area produced by the arrangement and layering; and,
- cost effectiveness of each arrangement.

2. Calculate the surface area of each package from the Predictions Chart. The dimensions of 1 orange triangle are 2 inches by 1 inch by $\sqrt{3} \approx 1.73$ inches and ≈ 0.375 inch. Record your calculations and the surface area for each arrangement in the Results Chart below.

Materials

- *TesselTiles: 24 orange triangles for each group of 4–6 students*
- *TesselTiles Template: 1 for each student group*
- *Inch Ruler: 1 for each student*
- *Calculator: 1 for each student group*

Results Chart: Packaging Arrangements for 24 Orange Right Triangles

Candy Layers		Surface Area Calculations	Total Surface Area (in.^{2})
Single	1		
	2		
	3		
	4		
Double	5		
	6		
	7		
Triple	8		
	9		
Quadruple	10		
	11		

Discussion Questions

1. Examine your Results Chart. Which arrangement produces the most cost-effective box for 24 orange right triangle-shaped candies?

2. Compare your rankings in your Predictions Chart with your Results Chart. How do your predictions compare with your results?

3. Review your Results Chart. Describe patterns for those arrangements that produced the smaller total surface areas.

4. Do you think the most cost-effective arrangement found in this activity would work for every TesselTiles™ shape? Explain.

5. **a.** Of the TesselTiles shapes, is the orange right triangle the best choice for a candy shape if packaging cost is the most important factor? If so, explain why. If not, which shape (or shapes) would be better?

b. Are there any general recommendations concerning candy shape and packing arrangements that you could write to the Sweet Acre Candy Company officials? Explain.

Further Investigations

1. The Sweet Acres Candy Company would like to manufacture a 10-pack candy box that contains 2 brown pentominoes and 8 of another candy shape. It asks for your advice on which candy shape they should use with the pentominoes.

a. Use the TesselTiles Template to determine which other shape would be best.

b. Find the dimensions of the most cost-effective rectangular box for your 10-pack candy assortment.

c. Find the surface area of the most cost-effective rectangular box for your 10-pack candy assortment.

Justify your thinking.